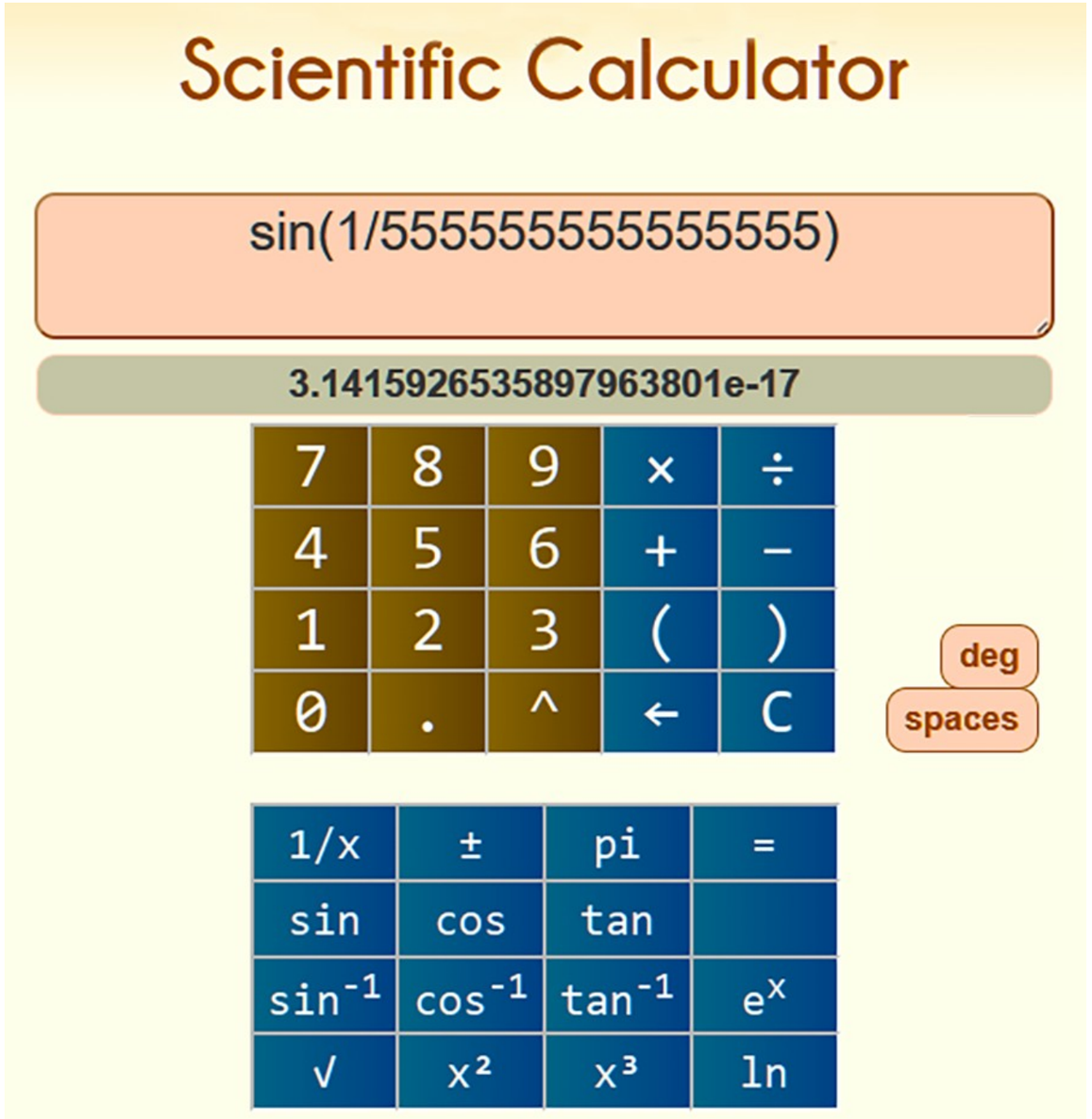


A puzzle:

← I typed this simple thing into this online calculator and
← got *this* as the result.

Why do the digits of π appear?

Note: *it's not that hard, so try to figure it out without Google <u>and</u> without using a calculator.*

(*Slide rules are OK.*)

More Fun With π

Michael Keith

March 2026

Digits

Algorithms

Approximations

Recreations

Pilish

Memorization

The First 768 Digits

$768 = 3 \times 2^8$

314159
265358979323846
2643383279502884197169
99375105820974944592307816
406286208998628034825342117 0
6798214808651328230664709384460 9
55058223172535940812848111745028 41
02701938521105559644622948954930 38
196442881097566593344612847564823378
678316527120190914564856692346034861
045432664821339360726024914127372458 70
066063155881748815209209628292540917 15
364367892590360011330530548820466521 38
414695194151160943305727036575959195 30
921861173819326117931051185480744623 79
962749567351885752724891227938183011 94
91298336733624406566430860213949463 9
52247371907021798609437027705392171 7
62931767523846748184676694051320 00
56812714526356082778577134275778 96
09173637178721468440901224953430
1465495853710507922796892589
235420199561121290219608 64
034418159813629774771 3
09960518707211 34
999999

What's this? (8 x 7 = 56 characters)

V	,	I	=	0	,	5	*
*	5		W	H	I	L	E
I	:	V	=	V	*	I	/
/	(	2	*	I	+	1	)
+	2	*	1	0	*	*	7
6	7	;	I	-	=	1	
P	R	I	N	T	(	V	)

What's this? (8 x 7 = 56 characters)

V	,	I	=	0	,	5	*
*	5		W	H	I	L	E
I	:	V	=	V	*	I	/
/	(	2	*	I	+	1	)
+	2	*	1	0	*	*	7
6	7	;	I	-	=	1	
P	R	I	N	T	(	V	)

```
v,i = 0,5**5
while i: v = v * i // (2*i + 1) + 2*10**767; i-=1
print(v)
```

How it Works

$$\pi = \sum_{n=0}^{\infty} \frac{(n!)^2\ 2^{n+1}}{(2n+1)!} \quad \text{(Euler, 1748)}$$

$$\frac{\pi}{2} = \sum_{n=0}^{\infty} \frac{n!}{(2n+1)!!} = 1 + \frac{1}{3} + \frac{1\cdot 2}{3\cdot 5} + \frac{1\cdot 2\cdot 3}{3\cdot 5\cdot 7} \cdots$$

$$\pi = 2 + \frac{1}{3}\left(2 + \frac{2}{5}\left(2 + \frac{3}{7}\left(2 + \frac{4}{9}(\cdots)\right)\right)\right)$$

- This series converges quite rapidly, unlike (for example) the Leibniz series $\pi = 4 - 4/3 + 4/5 - 4/7 \ldots$

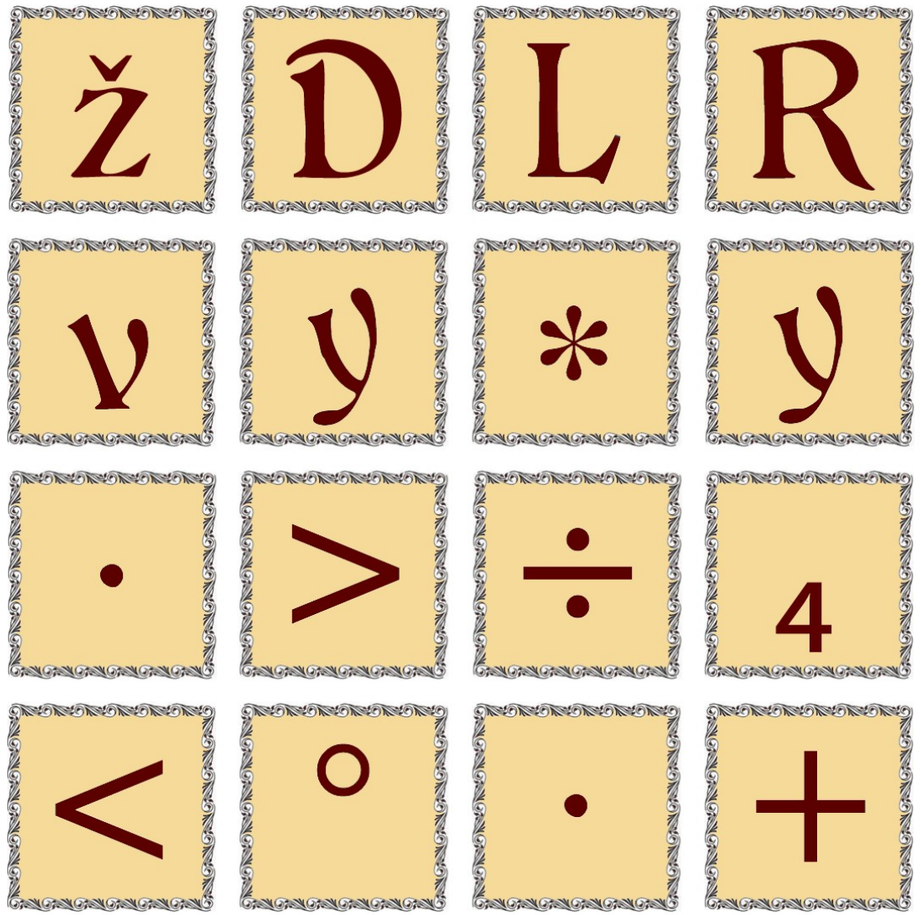
- Multiply equation by any 10^{k-1} to get the first *k* digits of π *using integer arithmetic*, calculating from right to left (= inside out)

16 characters (!) in the 05AB1E language

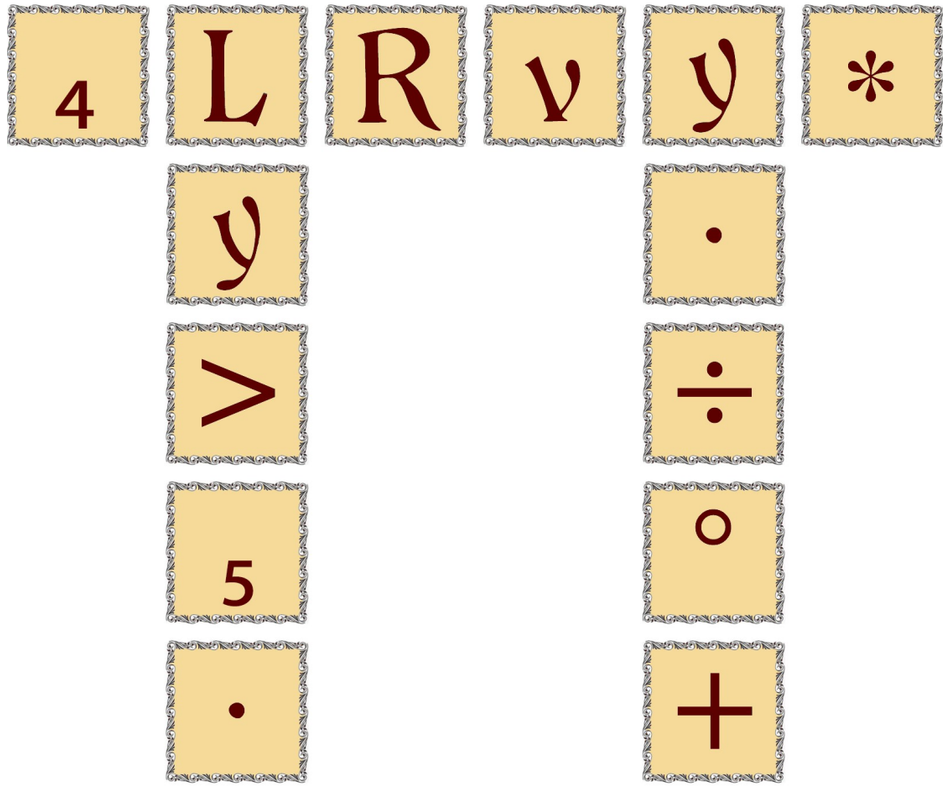
produces 1000 digits of π

```
     (Top of stack is a variable, P, initially 0)
žD       the number 4096
LR       make a reverse list: [4096..1]
v        loop over 4096..1 (implicit index 'y')
y*       P*y
y·>÷     divide by (y*2 + 1).
„<°·+    add (2 x 10^999), put result in P
         implicit: end of loop
         implicit: print number on top of stack
```

```
P = 0
for i = 4096 down to 1
  P = P*i / (2*i + 1) + (2 x 10^999)
print P
```



A 14 character “π” → 256 digits of π



produces

314159265358979323846264338327950288419716939937510582097494459230781640628620899862803482534211706798214808651328230664709384460955058223172535940812848111745028410270193852110555964462294895493038196442881097566593344612847564823378678316527120190914564

An Elegant Approximation

$$1\ 3\ 5 \downarrow 113355 \downarrow 113\overline{)355}$$

$$
\begin{array}{r}
3.141592 \\
113\overline{)355} \\
339 \\
\hline
160 \\
113 \\
\hline
470 \\
452 \\
\hline
180 \\
113 \\
\hline
670 \\
565 \\
\hline
1050 \\
1017 \\
\hline
330 \\
226 \\
\hline
104
\end{array}
$$

$$
3 + \cfrac{1}{7 + \cfrac{1}{15 + \cfrac{1}{1 + \cfrac{1}{292 + \cfrac{1}{1 + \cdots}}}}}
$$

$$= [3,\ 7,\ 15,\ 1,\ 292,\ 1\ldots]$$

More Approximations

Define *efficiency* $E = \dfrac{\text{\# of correct digits of } \pi}{\text{\# of digits in the expression}}$

$\dfrac{355}{113} = 3.141592920\ldots$ $\quad E = 7/6 \quad \approx 1.2$

$\sqrt{\sqrt{97+\dfrac{9}{22}}} = 3.14159265258264\ldots$ $\quad E = 9/5 \quad = 1.8$

The cont. frac. for $\pi^4 - 97$ *is* [0, 2, 2, 3, 1, 16539, 1...]

$\dfrac{\ln\left(640320^3+4!+6!\right)}{\sqrt{163}} = 3.14159265358979323846264338327\mathbf{726}\ldots$ $\quad E = 31/12 \quad \approx 2.6$

Pilish

"Pilish" is English with an added constraint:

The number of letters in each word follows the digits of π.

"How I need a drink (alcoholic, of course)

3 1 4 1 5 9 2 6

after the heavy lectures involving quantum computing."

5 3 5 8 9 7 9

- Very much in the spirit of Oulipo, the (mostly) French literary group

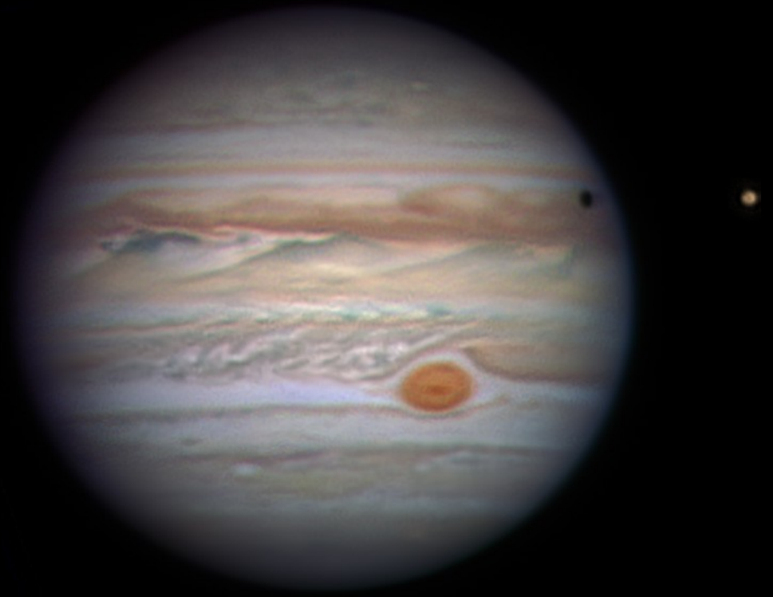
Piem #1

But a time I spent wandering in gloomy night,
   Yon tower, tinkling chimewise, loftily opportune;
Out, up, and together came twelve to Sunday rite,
   And one serenely off to observe plenilune.

Joseph Shipley (1960) &
Michael Keith (later)

(31 digits)

Piem #2



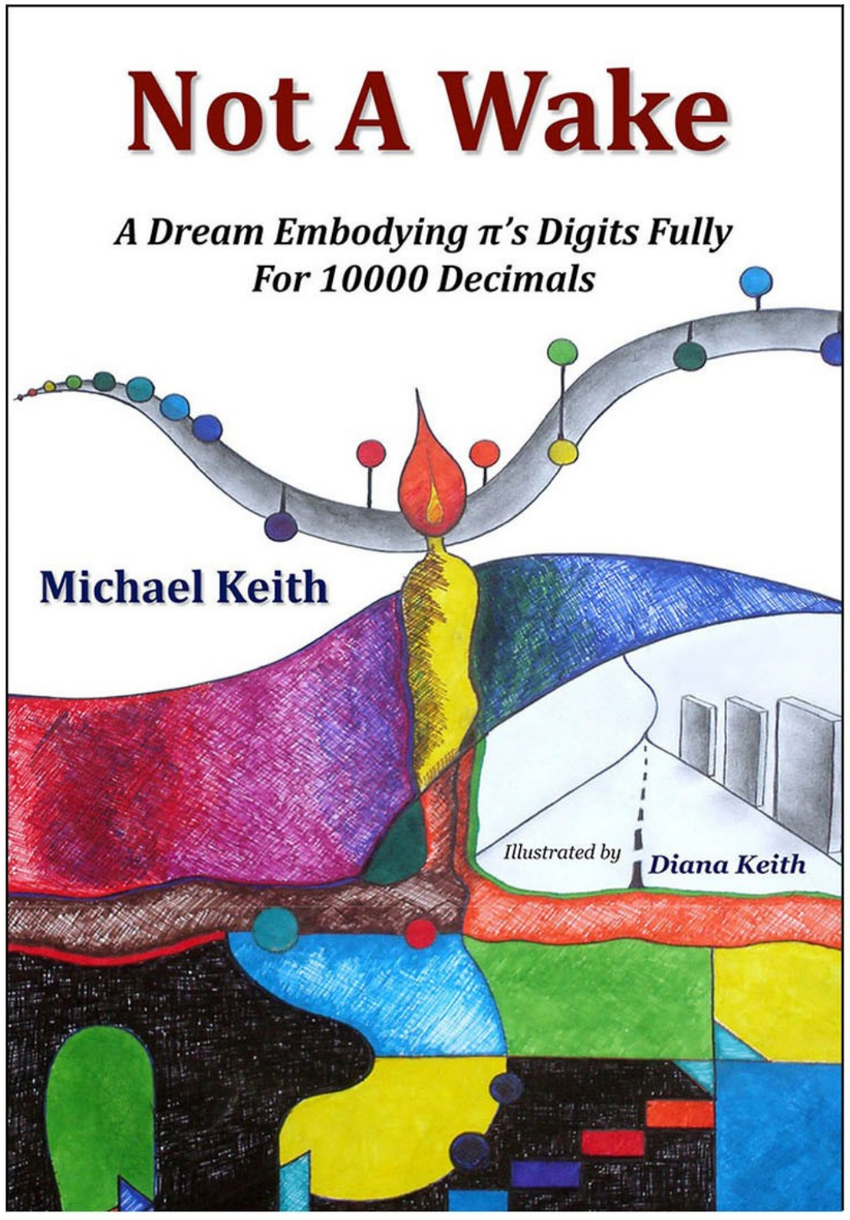
See a moon – a globe brilliant in flight –
Cross the great Jupiter's celestial ellipse;
Computers use pi – one constant they recite –
To create data now for tracking sun at eclipse.

Howard Bergerson (1965) &
Michael Keith (later)

(30 digits)

Dealing with 0's – and one more handy rule

31415926535	Now I fall, a tired suburban in liquid under the trees
8979323846	Drifting alongside forests simmering red in the twilight over Europe.
2643383279	So scream with the old mischief, ask me another conundrum
502884197	About bitterness of possible fortunes near a landscape Italian.
169399375	A little happiness may sometimes intervene but usually fades:
1058209 74	A missionary cries, striving to understand worthless, tedious life.
9445923078	Monotony's lost amid ocean movements as the bewildered sailors hesitate.
164062862 08	I become salt, submerging people in dazzling oceans of enshrouded unbelief.
998	Christmas ornaments conspire.
628034	Beauty is, somewhat inevitably now, both
82534211	Feelings of faith and eyes of **rationalism**.



Now I fall, a tired suburbian in liquid under the trees
Drifting alongside forests simmering red in the twilight over Europe.
So scream with the old mischief, ask me another conundrum
About bitterness of possible fortunes near a landscape Italian.
A little happiness may sometimes intervene but usually fades.
A missionary cries, striving to understand worthless, tedious life.
Monotony's lost amid ocean movements
As the bewildered sailors hesitate. I become salt,
Submerging people in dazzling oceans of enshrouded unbelief.
Christmas ornaments conspire.
Beauty is, somewhat inevitably now, both
Feelings of faith and eyes of rationalism.

Blinded delusional horses stumble;
Facetious nonsense is a dark, secluded tabernacle.
Comfort's buried: bleed a bit as antidote. Is one recovering?
Verily, octopi sing:
Burning choristers accompany the mournful song.
Don't ponder constantly – existence waits,
Among sunsetting tones, bringing it to you.
A wedding of birds and boars compounds with disloyalty,
Devising contemporary treasons.
This morning's displeasure: a badger's life ended,
Frightened to roadkill when a procession of hearses approached.
I whispered the profound truth of symmetrical restraints:
Untie every chain, sacrifice belief, free each beggar,
Go to everybody with peaceful, beautiful hands.

From stairways the multitudes fly downward,
A pointless heaven-like hell to conceive together.
A tourniquet-enwrapped servant walks beside Dover's beach,
Creatures cut the skin deep within a so-infinite void.

Accidental Pilish (8 digits)

Captain Cook's Journal During His First Voyage Round The World (ca. 1772)

From the entry for **June 14, 1769**:

...Contrary to the opinion of everybody, I would not suffer them to be fired upon, for this would have been putting it in the power of the Centinels to have fired upon them upon the most slitest occasions, as I had before experienced. And I have a great Objection to firing with powder only amongst People who know not the difference, for by this they would learn to despise fire Arms and think their own Arms superior, and if ever such an Opinion prevailed they would certainly attack you, the Event of which might prove as unfavourable to you as them...

The World Record (10 digits, two known examples)

From a model railroading online forum, written Dec 2009:

3 1 4 15 9 2 6 5 3

David has given you a very straightforward procedure to follow. Since you have not done any CV programming…

- found by M. Keith, Apr 2010

From a diabetes online forum, written Aug 2013:

3 1 4 1 5 9 2 6 5 3

I wake up when LO and I have a small container of orange juice and a package of peanut butter crackers.

- found by M. Practice, Aug 2023


MAY I HAVE A LARGE CONTAINER OF POTATO SALAD?


For a time, I had this thought...

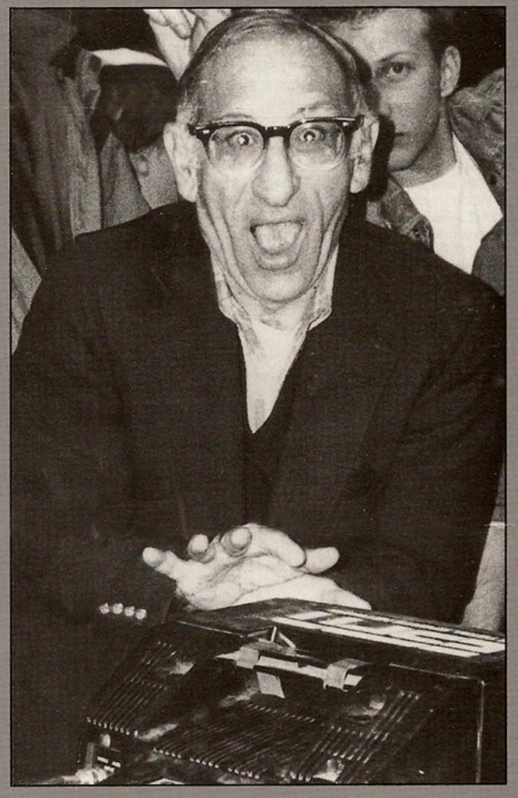

Men, a ball, a pitch.


How I wish I could have a different dad.

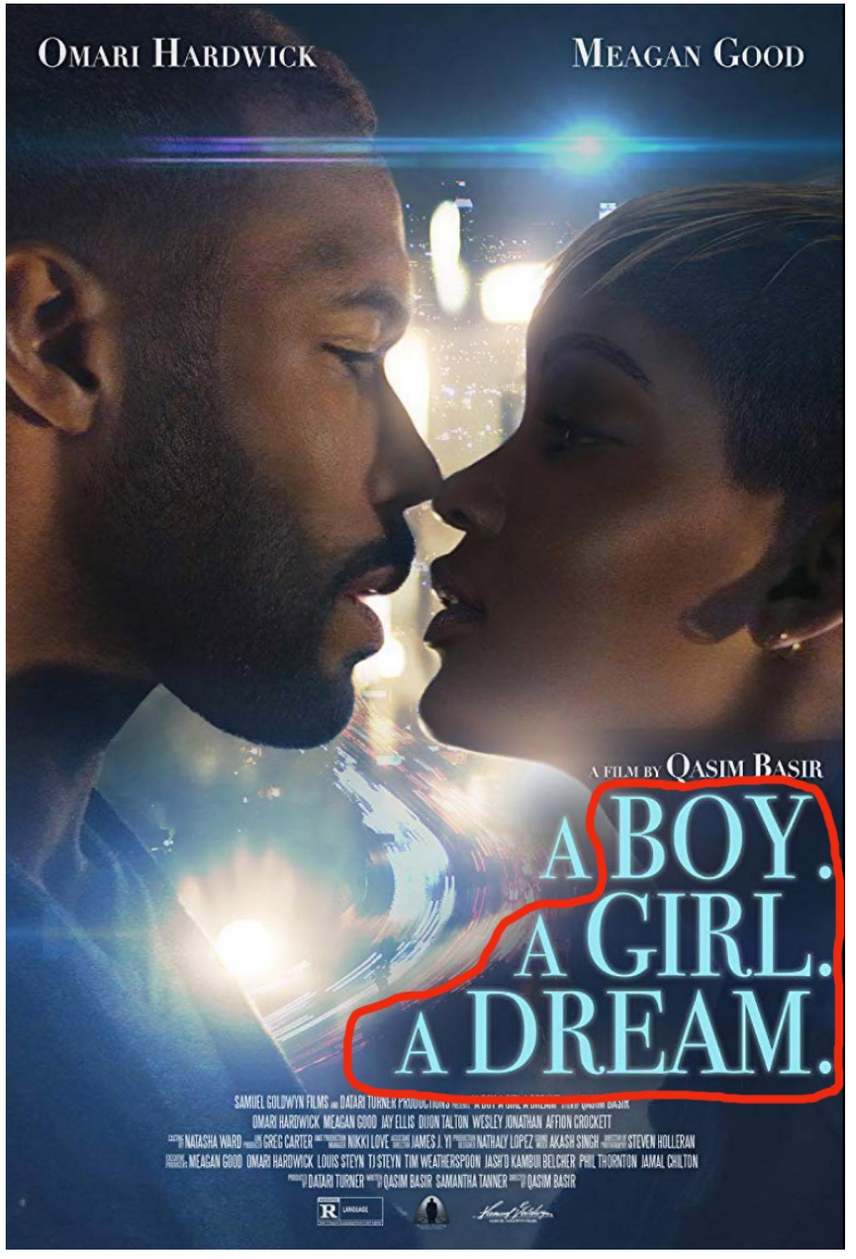

And I have a great example.

A Boy, A Ball, A Dream

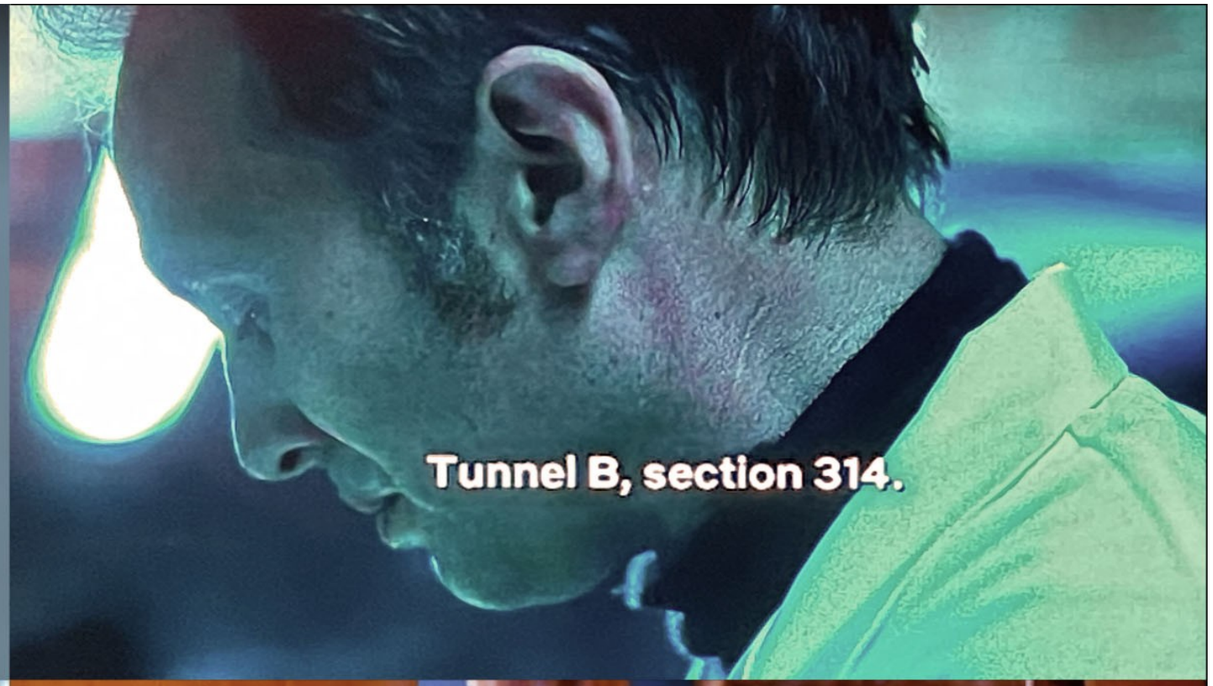
The amazing life of the world's most devoted basketball junkie, Tom J. "Toody" Cirincione

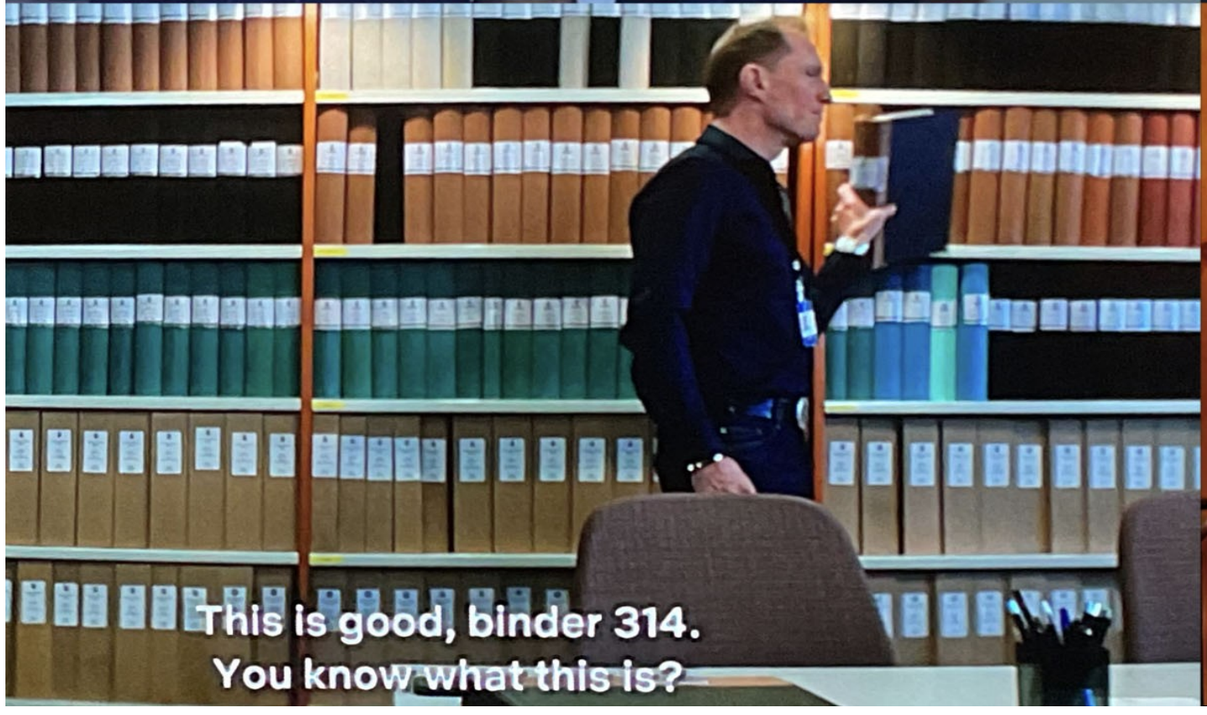


by Chris Roche




It's 3.14 percent.


Tunnel B, section 314.


This is good, binder 314.
You know what this is?


315
314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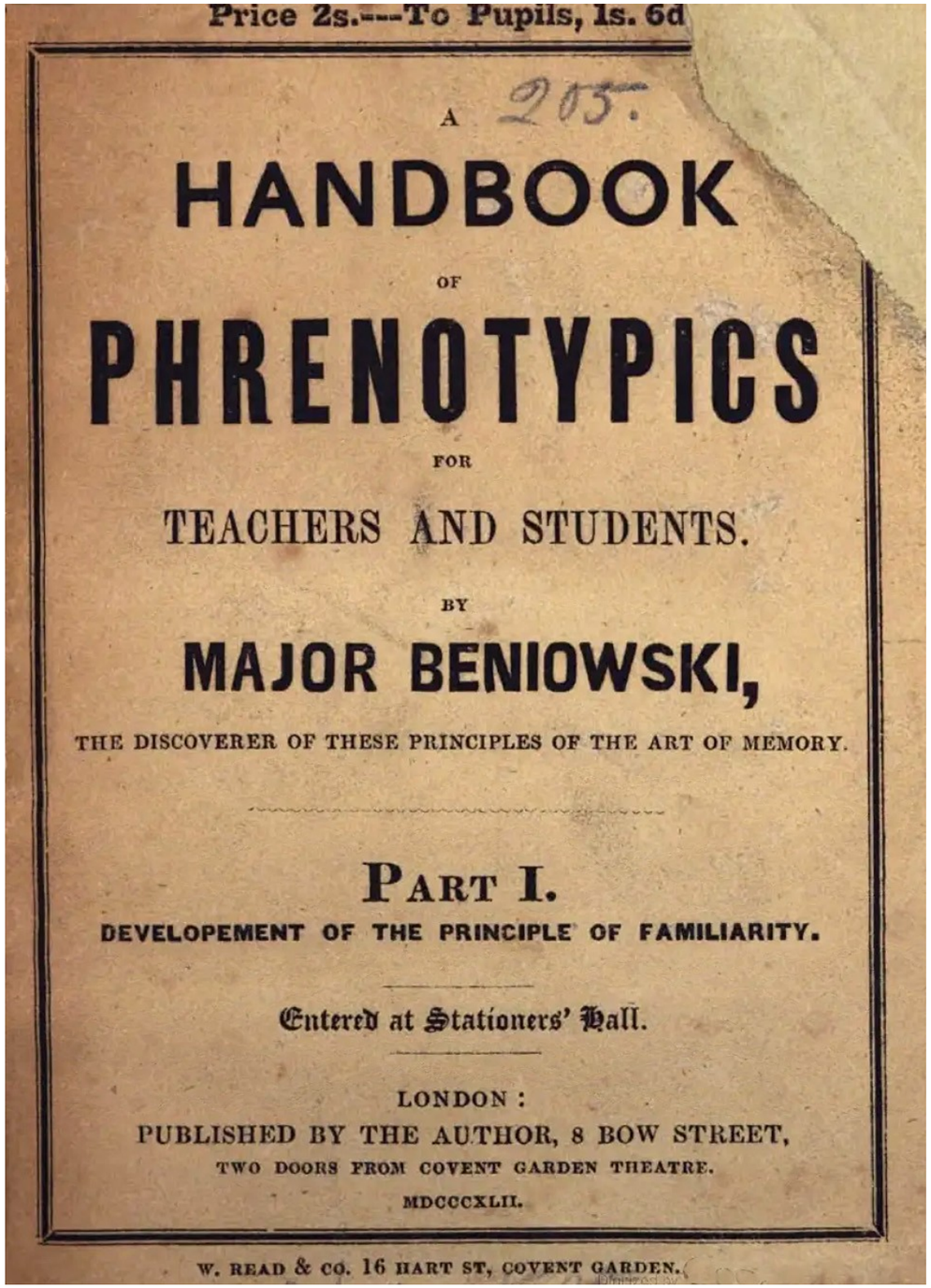
Memorizing π: A Deep Dive

Using the Literal Major System + a “Memory Palace”

6	All locations combined, with a path through them	My house
5	Locations for the objects below - should be very familiar places	Dining Rm. table
4	Phrases or sentences made from the words below - should be *visually rich* and *personal*	The Earl Grey flag
3	One word for each digit group - letters ↔ digits via a defined mapping	856 → flag
2	Groups of the digits below - group them to optimize the effectiveness of #3 and #4	1 45 64 856
1	The digits to be memorized	14564856...

The Original Major System (1842)



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A

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OF

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FOR

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BY

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THE DISCOVERER OF THESE PRINCIPLES OF THE ART OF MEMORY.

PART I.

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The Literal Major System

- The standard “Major System” is *phonetic*. Making it ***literal*** is more precise.

0	z s	zero
1	t d	T has 1 vertical line
2	n	n has 2 vertical lines
3	m	m has 3 vertical lines
4	r	four
5	L	L = 50
6	G j	G looks like 6
7	k c	k is composed of two 7’s
8	f v	*f* in cursive has two loops, like 8
9	b p	b upside down looks like 9

16 key letters

- H, Q, W, X and the vowels AEIOUY don’t represent a digit
  (they’re just used to help create words)

10 free letters


LAWSON
The Earl Gray flag
1 45 64 85 6


Japan: More gas, more Fuji
6 9 2 3 4 6 0 3 4 8 6
LAWSON
The Earl Gray flag
1 45 64 85 6


The serial-rhyming,
1 0 4 5 4 3 26
groovin'
64 8 2
Tommy Bahama
1 33 9 3
Japan: More gas, more Fuji
6 9 2 3 4 6 0 3 4 8 6
LAWSON
The Earl Gray flag
1 45 64 85 6


The serial-rhyming,
1 0 4 5 4 3 26
groovin'
64 8 2
Tommy Bahama
1 33 9 3
Japan: More gas, more Fuji
6 9 2 3 4 6 0 3 4 8 6
LAWSON
Gas exchanges
6 0 7 26 0
enrapture Ethan Hawke
24 91 4 1 2 7
The Earl Gray flag
1 45 64 85 6